$$X_t = \sigma(t) f(t) \int_0^t \frac{dW_s}{f(s)}$$

$$m(t) = 0, \qquad a(s,t) = \sigma(s) f(s) \sigma(t) f(t) \int_0^{s \wedge t} \frac{dr}{f(r)^2}$$

$$X_t = \int_0^t \frac{(\sigma(s) f(s))'}{\sigma(s) f(s)} X_s \, ds + \int_0^t \sigma(s) \, dW_s$$

Casi particolari

**Brownian bridge**

$$\sigma(t) = 1, \qquad f(t) = 1 - t$$

$$X_t = (1-t) \int_0^t \frac{dW_s}{1-s}$$

$$m(t) = 0, \qquad a(s,t) = s \wedge t - st$$

$$p(s,x;t,y) = \frac{1}{\sqrt{2\pi \frac{1-t}{1-s}(t-s)}} \exp\Big( -\frac{(y - \frac{1-t}{1-s}x)^2}{2\frac{1-t}{1-s}(t-s)}\Big)$$

$$\frac{\partial p}{\partial s} = -\Big(\frac{1}{2}\frac{\partial^2 p}{\partial x^2} - \frac{x}{1-s}\frac{\partial p}{\partial x}\Big) \quad \text{backward Kolmogorov eqn.}$$

$$\frac{\partial p}{\partial t} = \frac{1}{2}\frac{\partial^2 p}{\partial y^2} + \frac{\partial}{\partial y}\Big(\frac{y}{1-t}\, p\Big) \quad \text{forward Kolmogorov eqn. (Fokker-Planck)}$$

$$X_t = -\int_0^t \frac{1}{1-s} X_s \, ds + W_t$$

**Ornstein-Uhlenbeck**

$$\sigma(t) = \sigma, \qquad f(t) = e^{\lambda t}$$

$$X_t = \sigma \int_0^t e^{\lambda(t-s)} dW_s$$

$$a(s,t) = \frac{\sigma^2}{2\lambda}(e^{\lambda(s+t)} - e^{\lambda|t-s|})$$

$$p(s,x;t,y) = \frac{1}{\sqrt{2\pi \frac{\sigma^2}{2\lambda}(e^{2\lambda(t-s)} - 1)}} \exp\Big( -\frac{(y - e^{\lambda(t-s)}x)^2}{2\frac{\sigma^2}{2\lambda}(e^{2\lambda(t-s)} - 1)}\Big)$$

$$\frac{\partial p}{\partial s} = -\Big(\frac{1}{2}\sigma^2 \frac{\partial^2 p}{\partial x^2} + \lambda x \frac{\partial p}{\partial x}\Big) \quad \text{backward Kolmogorov eqn.}$$

$$\frac{\partial p}{\partial t} = \frac{1}{2}\sigma^2 \frac{\partial^2 p}{\partial y^2} - \frac{\partial}{\partial y}\big(\lambda y\, p\big) \quad \text{forward Kolmogorov eqn. (Fokker-Planck)}$$

$$X_t = \lambda \int_0^t X_s\, ds + \sigma\, W_t$$

**Ancora un Ornstein-Uhlenbeck (stazionario)**

$$\sigma(t) = \sigma, \qquad f(t) = e^{-\lambda t}, \quad \lambda > 0$$

$$X_t = X_0\, e^{-\lambda t} + \sigma \int_0^t e^{-\lambda(t-s)} dW_s$$

dove $X_0$ ha come legge $N(0, \frac{\sigma^2}{2\lambda})$ ed è indipendente dal processo di Wiener $W_t$

$$m(t) = 0, \qquad a(s,t) = \frac{\sigma^2}{2\lambda} e^{-\lambda\,|t-s|}$$

$$X_t = X_0 - \lambda \int_0^t X_s\, ds + \sigma\, W_t$$

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$$\dot{X}_t = -\frac{1}{1-t} X_t + \dot{W}_t, \qquad dX_t = -\frac{1}{1-t} X_t\, dt + dW_t$$

$$\dot{X}_t = \lambda\, X_t + \sigma\, \dot{W}_t, \qquad dX_t = \lambda\, X_t\, dt + \sigma\, dW_t$$

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$$dX_t = b(t, X_t)\, dt + \sigma(t, X_t)\, dW_t$$

$$Au = \frac{1}{2}\sigma(t,x)^2\, \frac{\partial^2 u}{\partial x^2} + b(t,x)\, \frac{\partial u}{\partial x}$$

$$Bv = \frac{1}{2}\, \frac{\partial^2}{\partial x^2}(\sigma(t,x)^2\, v) - \frac{\partial}{\partial x}(b(t,x)\, v)$$