$$a_0, \quad a_1, a_2, a_3, \dots \quad b_1, b_2, b_3, \dots \qquad a_i, b_i \in \mathbb{R}$$

$$s(x) := \tfrac{a_0}{2} + \sum_{k=1}^{\infty} \left[a_k \cos(kx) + b_k \sin(kx) \right]$$

$$s_n(x) := \tfrac{a_0}{2} + \sum_{k=1}^{n} a_k \cos(kx) + b_k \sin(kx)$$

$$s_n \xrightarrow{?} s$$

$$x \mapsto f(x)$$

$$a_0, \quad a_1, a_2, a_3, \dots \quad b_1, b_2, b_3, \dots \qquad a_i, b_i \in \mathbb{R}$$

$$f(x) = \tfrac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx)$$

$$\boxed{\begin{aligned} a_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, \cos(kx)\, dx, \quad k = 0, 1, 2, 3, \dots \\ b_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, \sin(kx)\, dx, \quad k = 1, 2, 3, \dots \end{aligned}}$$

$$f(x) \stackrel{?}{=} s(x) := \tfrac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx)$$

$$\frac{a_k}{\sqrt{a_k^2+b_k^2}}, \qquad \frac{b_k}{\sqrt{b_k^2+b_k^2}}, \quad k=1,2,3,\dots$$

$$\cos\varphi_k = \frac{a_k}{\sqrt{a_k^2+b_k^2}}, \qquad \sin\varphi_k = \frac{b_k}{\sqrt{b_k^2+b_k^2}}, \quad k=1,2,3,\dots$$

$$A_k = \sqrt{a_k^2+b_k^2}$$

$$\tfrac{a_0}{2} + \sum_{k=1}^{\infty} a_k\cos(kx) + b_k\sin(kx) = \tfrac{a_0}{2} + \sum_{k=1}^{\infty} A_k\Big[\cos\varphi_k\cos(kx) + \sin\varphi_k\sin(kx)\Big]$$

$$\tfrac{a_0}{2} + \sum_{k=1}^{\infty} a_k\cos(kx) + b_k\sin(kx) = \tfrac{a_0}{2} + \sum_{k=1}^{\infty} A_k\cos(kx-\varphi_k)$$

$$A_0 := \frac{|a_0|}{2}, \qquad \begin{cases} \varphi_0 = 0, & a_0 > 0 \\ \varphi_0 = \pi & a_0 < 0 \end{cases}$$

$$\tfrac{a_0}{2} + \sum_{k=1}^{\infty} a_k\cos(kx) + b_k\sin(kx) = \sum_{k=0}^{\infty} A_k\cos(kx-\varphi_k)$$